STARKWARE STARK 101

Stark 101: Part 1

Statement, LDE and Commitment

FibonacciSq

(Fibonacci Square)

STARKWARE

FibonacciSq (Fibonacci Square)

FibonacciSq: $a_{n+2} = a_{n+1}^2 + a_n^2$

- Represented as: $a_0, a_1, a_2, a_3, \ldots$
- Determined by first two elements
- Example:
  - 1, 3, 10, 109, 11981, 143556242,...

Tiny Problem

a_{10}=10585384481491331545443435980195330168085
227108560824098919278258215839789697544114437
130080556524289168854586579782387518129922282
832261605608145523797747714827465842570005148
785265883367108772402086618503369319342561663
36593387070293738452872952783090264176685

FibonacciSq Mod Prime

FibonacciSq mod prime: $a_{n+2} = a_{n+1}^2 + a_n^2 \mod prime$

Example:

- 1, 3, 10, 109, 11981, 143556242,...

mod 7:

- 1, 3, 3, 4, 4, 4, ...

FibonacciSq Mod Prime

FibonacciSq mod prime: $a_{n+2} = a_{n+1}^2 + a_n^2 \mod prime$

- Example - mod 7:
  - 1, 3, 3, 4, 4, 4, ...

We use *prime* = $3 \cdot 2^{30} + 1 = 3221225473$

Finite field *F*

Statement

Statement to Prove

There is a number *x* such that:

For the FibonacciSq mod 3221225473 with

- $a_0 = 1$
- $a_1 = x$

we have $a_{1022} = 2338775057$

X = 3141592

STARK Protocol

STARK Protocol - Part I

- LDE - Low Degree Extension
- Commitment

Low Degree Extension

(LDE)

LDE in 3 Steps

1. Generate input
2. Interpolate
3. Extend

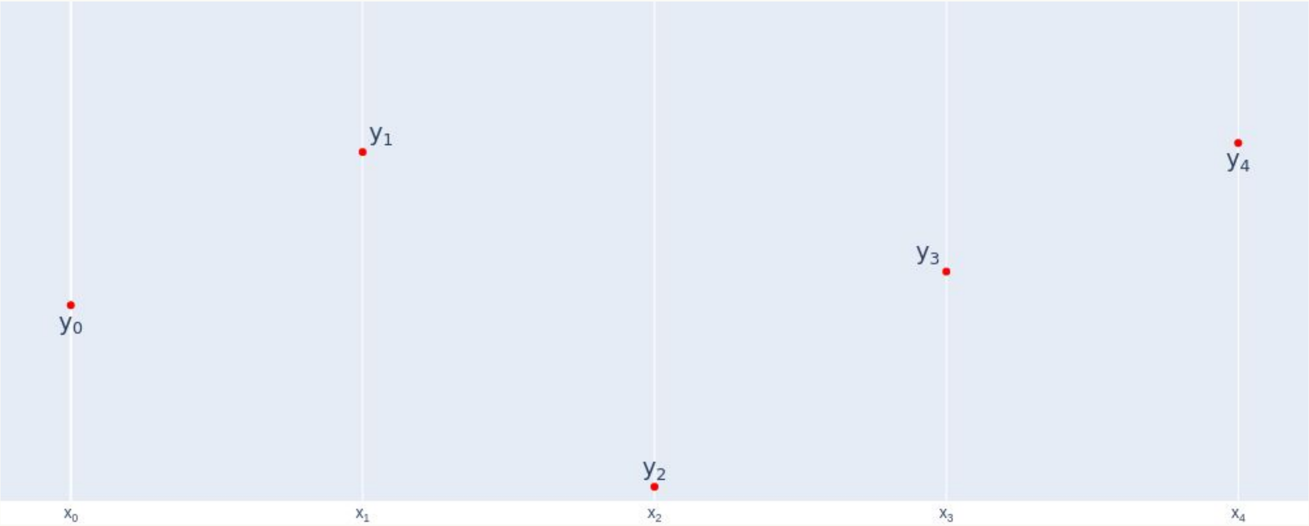
LDE - General

LDE Step 1 - Generate Input

Input: $y_0, y_1, y_2, y_3, y_4, \ldots$

Choose: $x_0, x_1, x_2, x_3, x_4, \ldots$

x	y
x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

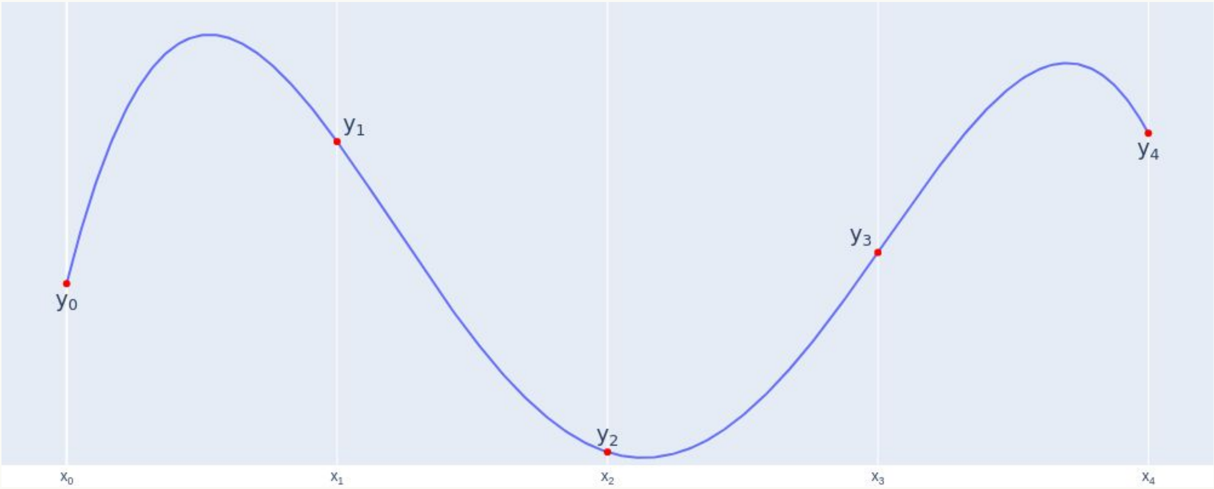


LDE Step 2 - Interpolate Polynomial

Interpolate a polynomial *f*:

For each $i : f(x_i)=y_i$

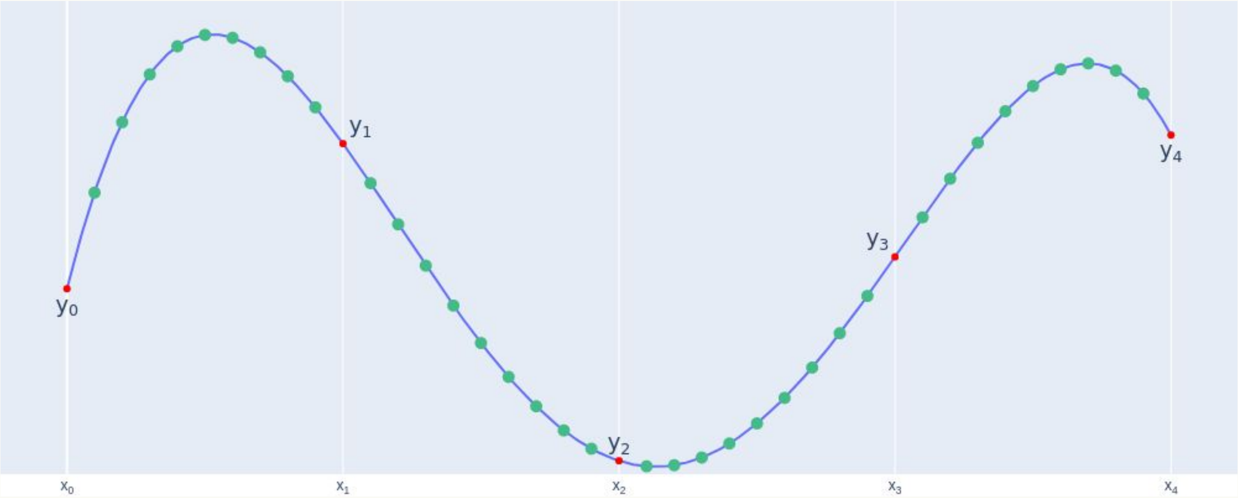
x	$f(x)$
x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4



LDE Step 3 - Extend

- Pick a larger evaluation domain $\{x_j`\}$
- Output: $\{f(x_j`)\}$

$x`$	$f(x`)$
$x`_0$	$f(x`_0)$
$x`_1$	$f(x`_1)$
$x`_2$	$f(x`_2)$
$x`_3$	$f(x`_3)$
...	...



LDE in STARK

LDE for STARK Step 1 - Generate Input

Input: $a_0, a_1, a_2, \ldots, a_{1022}$

The **Trace**

We choose: $1, g, g^2, g^3, \ldots, g^{1022}$

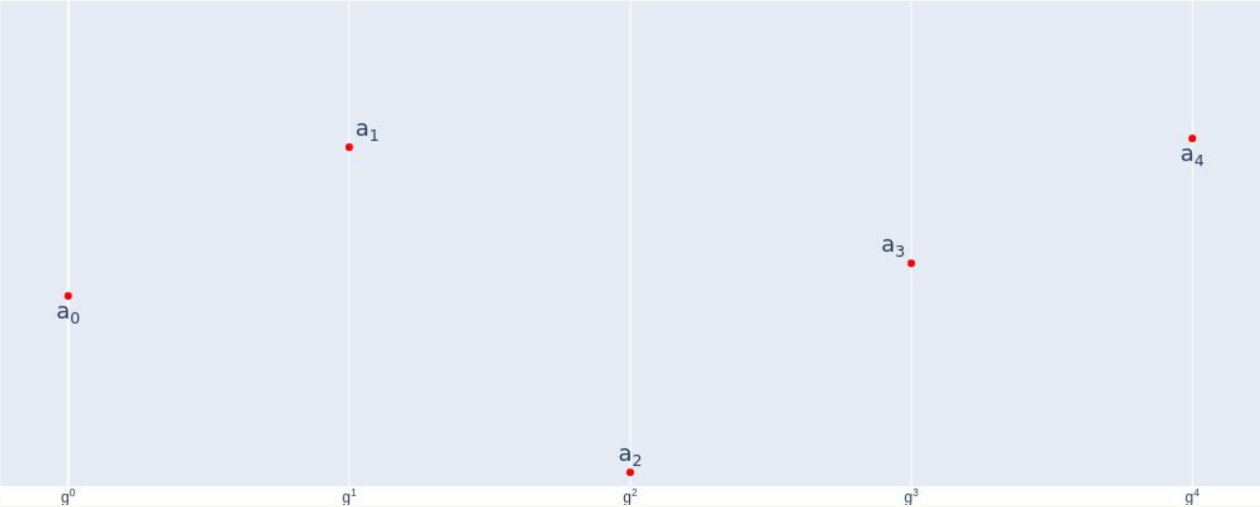
g - element from F

LDE for STARK Step 1 - Generate Input

Input: $a_0, a_1, a_2, \ldots, a_{1022}$

We choose: $1, g, g^2, g^3, \ldots, g^{1022}$

x	$f(x)$
g^0	a_0
g^1	a_1
g^2	a_2
...	...
g^{1022}	a_{1022}

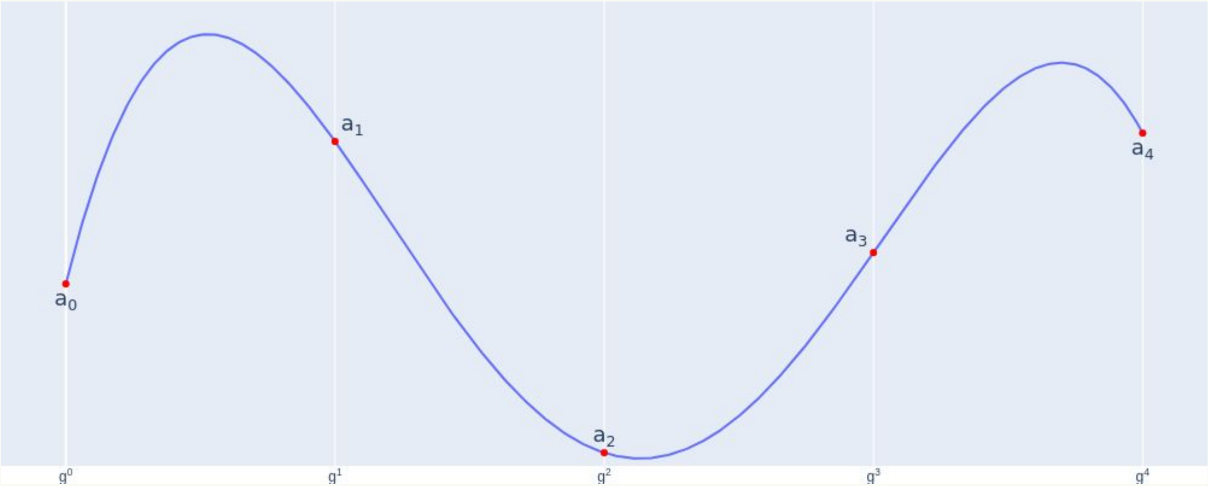


LDE for STARK Step 2 - Interpolate Poly

Interpolate a polynomial *f*:

$$\text{for each } i : f(g^i)=a_i$$

x	$f(x)$
g^0	a_0
g^1	a_1
g^2	a_2
...	...
g^{1022}	a_{1022}



LDE for STARK Step 3 - Extend

- Pick a larger evaluation domain (8k)
- $\{x_i`\} = w, w \cdot h, w \cdot h^2, \ldots, w \cdot h^{8191}$

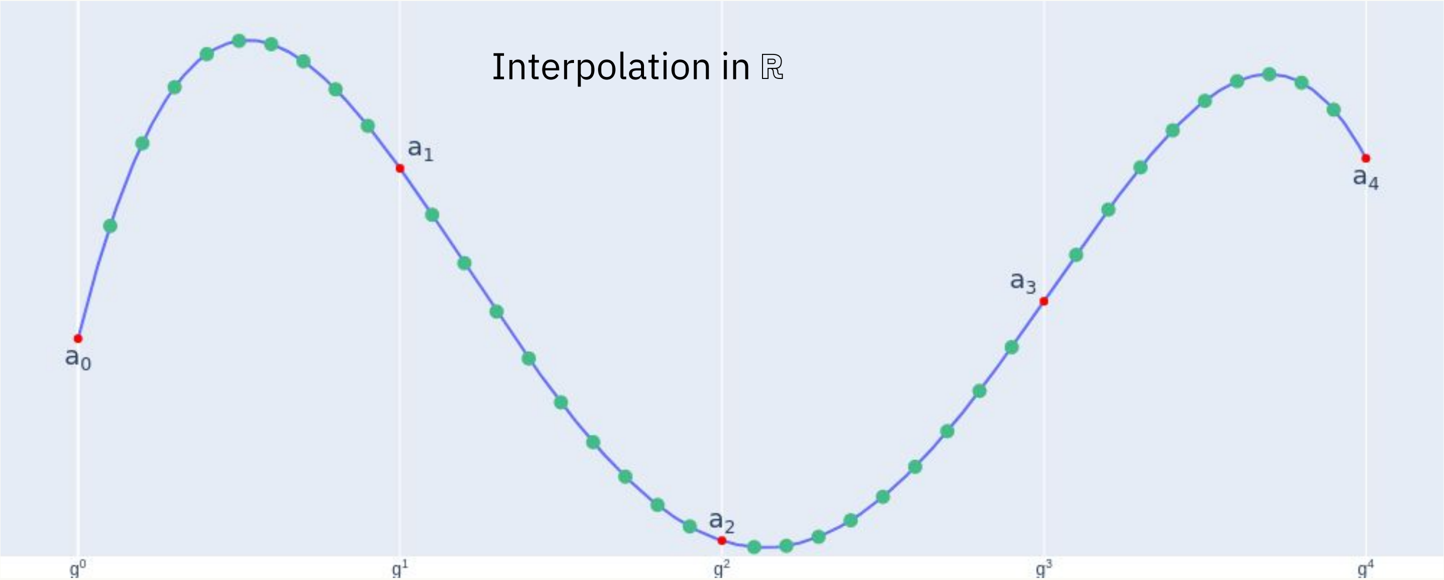
w, h - elements from F

- Result: $f(w), f(w \cdot h), f(w \cdot h^2), \ldots$

Reed-Solomon codeword

LDE for STARK Step 3 - Extend

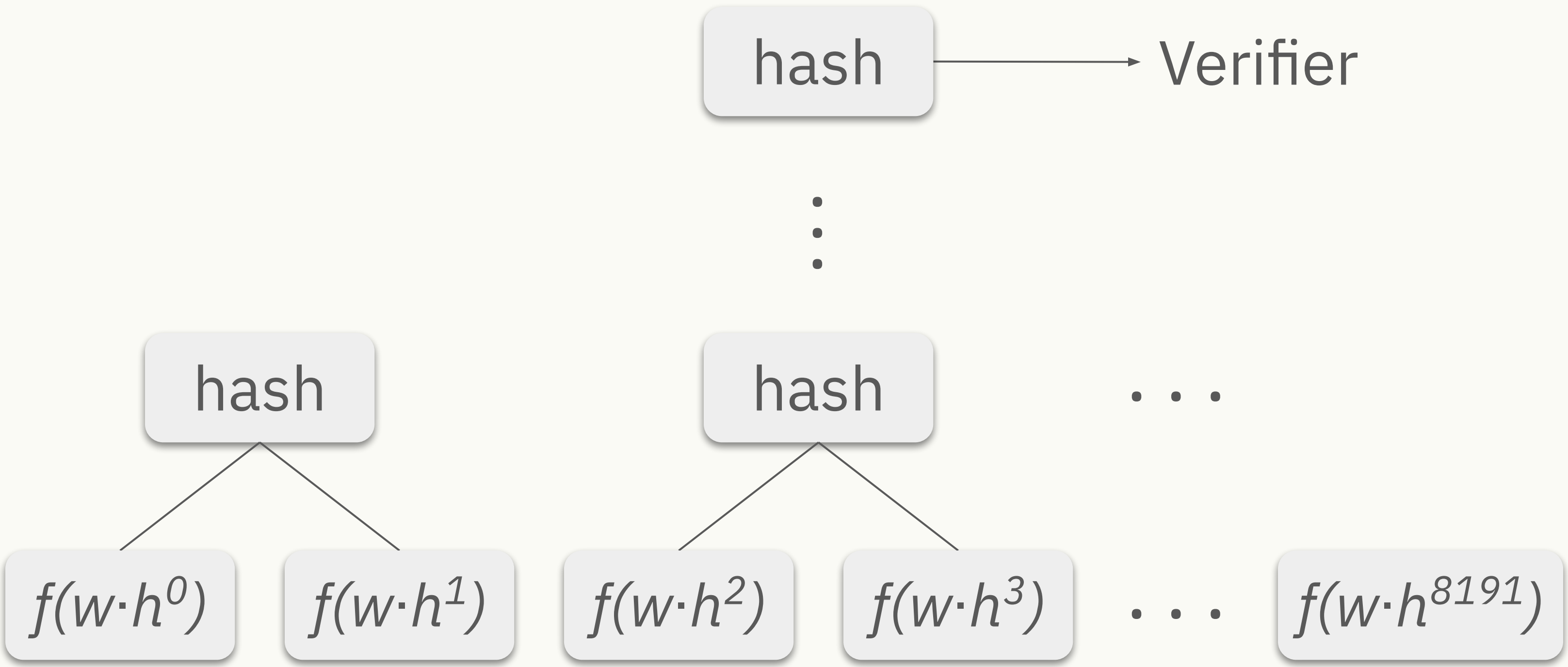
x	$f(x)$
$w \cdot h^0$	$f(w \cdot h^0)$
$w \cdot h^1$	$f(w \cdot h^1)$
$w \cdot h^2$	$f(w \cdot h^2)$
…	…
$w \cdot h^{8191}$	$f(w \cdot h^{8191})$



Commitment

Commit on LDE

Merkle Tree



Summary

- Statement
  - There is x s.t. a_{1022}= 2338775057 in FibonacciSq mod prime
- STARK protocol - part I:
  - LDE - Low Degree Extension
  - Commitment - Merkle Tree

What’s Next?

Part 2 - polynomial constraints

But first - coding.....

1) Trace, LDE

2) Commit LDE Trace.

google:

‘github stark 101’

Thank you