STARKWARE STARK 101

Stark 101: Part 3

FRI Commitment

Recap

Goal : prove a statement on FibonacciSq

- Trace in 1023 points
- Create *Trace* polynomial (Lagrange interpolation)
- Evaluate and commit on a larger domain

Recap

- 3 constraints on *f(x)*:

$$f(x) - 1 = 0 \quad , \text{for } x = 1$$

...

- 3 rational functions from the constraints:

$$p_0(x) = \frac{f(x) - 1}{x - g^0}$$

...

Recap

- **C**omposition **P**olynomial:

$$CP(x) = \alpha_0 \cdot p_0(x) + \alpha_1 \cdot p_1(x) + \alpha_2 \cdot p_2(x)$$

- Prover commits on CP
- Goal - show that CP is a **polynomial**
- CP is a **polynomial** $\longrightarrow$ All constraints satisfied

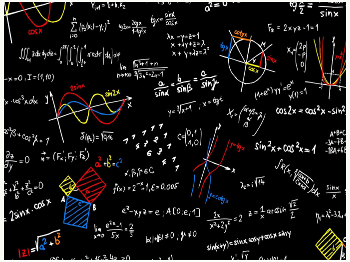
What Will We Do?

Goal:

Prove that CP is a **polynomial**

Instead:

Prove that CP is **close** to a **polynomial** of **low degree**



What is close?

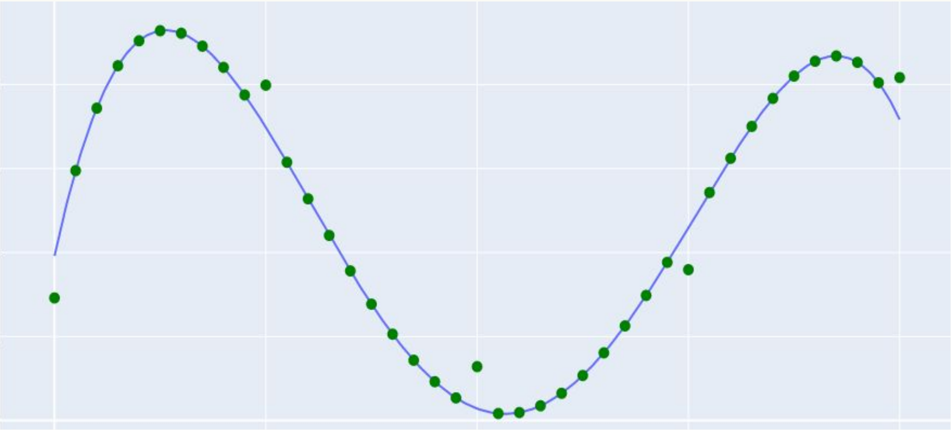
What is low degree?

Proximity to Polynomials

Distance (def):

Distance between a function $\boldsymbol{f}: D \rightarrow F$ *to a polynomial* $\boldsymbol{p}$:

$D(f,p)$:= # points $x \in D$ such that $f(x) \neq p(x)$



$D(\boldsymbol{f}, \boldsymbol{p}) = 5$

Proximity to Polynomials

Distance (def):

Distance between a function ***f***: $D \to F$ *to a polynomial* ***p***:

$D(f,p)$:= # points $x \in D$ such that $f(x) \neq p(x)$

Proximity

A function $f: D \to F$ is ***close*** *to a* polynomial p if: $D(f,p)$ is **small**

What Will We Do? - Reminder

Goal:

Prove that CP is **close** to a **polynomial** of **low degree**

How?


Trust me,
the commitment is close to
a low degree polynomial
gifs.com

FRI

Fast Reed-Solomon Interactive Oracle Proofs of Proximity

By Ben-Sasson, E., Bentov, I., Horesh, Y., & Riabzev, M.

https://eccc.weizmann.ac.il/report/2017/134/

FRI - Goal

Prover convinces verifier:

“The commitment is close to a low degree polynomial“

FRI - The Protocol

- Receive random β
- Apply the FRI operator
- Commit
- Lastly the prover sends the result

Do it repeatedly

FRI

- FRI operator - motivation
- FRI steps overview
- Deep into the FRI operator

FRI Operator

FRI Operator

Goal:

Prove that a function is close to a polynomial of a degree $< D$

↓ Applying the FRI operator

New Goal:

Prove that a **new** function is close to a **new** polynomial

↑ Half of the domain size (the **new** function)

↑ Degree $< D/2$ (the **new** polynomial)

FRI Operator - Example
Before applying FRI operator

- Prove:

A function is close to a polynomial of a degree < **1024**

where domain size = **8192**

FRI Operator - Example

~~Before~~ <u>After</u> applying FRI operator

- Prove:

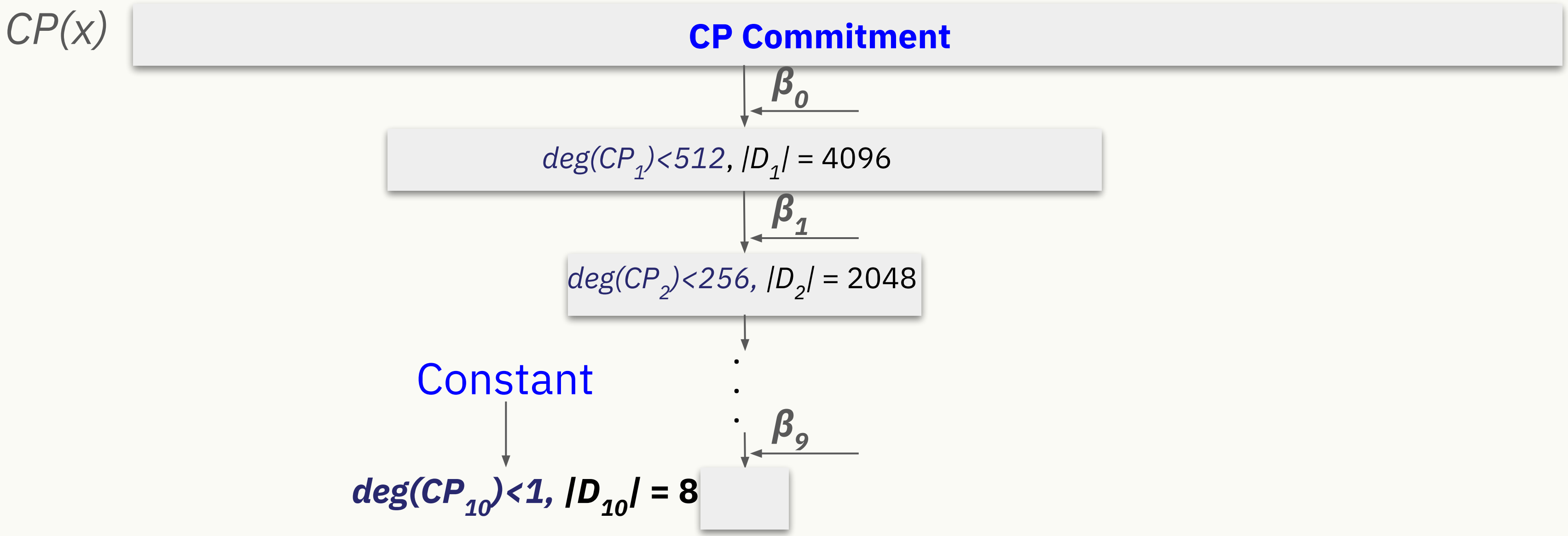
  A function is close to a polynomial of a degree < ~~**1024**~~ **512**

  where domain size = ~~**8192**~~ **4096**

FRI Steps Overview

FRI Steps Overview

Showing that *deg(CP)<1024, |D|=8192*



FRI Steps Overview

Showing that *deg(CP)<1024, |D|=8192*

CP(x)

CP Commitment

β_0

deg(CP_1)<512, |D_1| = 4096

β_1

deg(CP_2)<256, |D_2| = 2048

.
.
.

β_9

deg(CP_{10})<1, |D_{10}| **= 8**

Deep Into the FRI Operator

FRI Operator - How Does it Work?

- Split to even and odd powers

  $$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

  $$P_1(y) = g(y) + \beta h(y)$$

- Example:

  $P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$

  $g(x^2)$: $3x^4$ $2x^2$ 3

  $xh(x^2)$: $5x^5$ $7x^3$ x

FRI Operator - How Does it Work?

- Split to even and odd powers

$$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

$$P_1(y) = \mathbf{g(y)} + \beta h(y)$$

- Example:

$$P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$$

$g(x^2)$: $3x^4$, $2x^2$, 3

$g(\mathbf{y})$: $3\mathbf{y}^2$, $2\mathbf{y}$, 3

$xh(x^2)$: $5x^5$, $7x^3$, x

FRI Operator - How Does it Work?

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$$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

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- Example:

$$P_0(x) = 5x^5 + 3x^4 + 7x^3 + 2x^2 + x + 3$$

$g(y)$: $3y^2$ $2y$ 3

$xh(x^2)$: $5x^5$ $7x^3$ x

FRI Operator - How Does it Work?

- Split to even and odd powers

$$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

$$P_1(y) = g(y) + \beta \mathbf{h(y)}$$

- Example:

$$P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$$

$g(y)$		$3y^2$		$2y$		3
$xh(x^2)$	$5x^5$		$7x^3$		x	
$h(y)$	$5\mathbf{y}^2$		$7\mathbf{y}$		$\mathbf{1}$	

FRI Operator - How Does it Work?

- Split to even and odd powers

$$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

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- Example:

$$P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$$

$g(y)$ $3y^2$ $2y$ 3

$h(y)$ $5y^2$ $7y$ 1

FRI Operator - How Does it Work?

- Split to even and odd powers

$$P_0(x) = g(x^2) + xh(x^2)$$

- Get a random β

- Consider the new function:

$$P_1(y) = g(y) + \beta h(y)$$

- Example:

$$P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$$

$g(y)$		$3y^2$		$2y$		3
$h(y)$	$5y^2$		$7y$		1	

- $P_1(y) = 3y^2 +2y+3+\beta(5y^2+7y +1)$

$$= (3+5\beta)y^2+(2+7\beta)y+3+\beta$$

FRI - The Protocol - Reminder

- Receive random β
- Apply the FRI operator
- Commit
- Lastly the prover sends the result

constant

Do it repeatedly

deg(poly) < 1
where
domain size is 8

FRI - The Protocol - A Single Step

$$P_0(x) = 5x^5+3x^4+7x^3+2x^2+x+3$$

Degree bound = 6

$\mathbf{deg(P_1) \le deg(P_0)/2}$

Degree bound = 3

- Receives random β
- Apply the FRI operator
- Commit

$$P_1(y) = (3+5\beta)y^2+(2+7\beta)y+3+\beta$$

Thank you